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FEKETE-SZEGÖ INEQUALITY FOR ANALYTIC AND BI-UNIVALENT FUNCTIONS RELATED WITH HORADAM POLYNOMIALS

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Abstract: In this research article, by making use of Sălăgean differential operator, we introduce and investigate a new subclass of analytic and bi-univalent functions using the Horadam polynomial. We derive the coefficient estimate and obtain Fekete-Szegö inequality for functions in this subclass.

Keywords and Phrases: Sălăgean differential operator, Horadam polynomials, coefficient estimates, Fekete-Szegö inequality.

2020 Mathematics Subject Classification: 30C45, 30C50.

1. Introduction

Let $\mathcal{A}$ denote the class of all analytic functions f defined on the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$, which is normalized under the condition $f(0) = f'(0) = 1$ having the Taylor series expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \ z \in \mathbb{U}. \tag{1.1}$$

and $\mathcal{S}$, the class of functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$. Let the function f and g be analytic in $\mathbb{U}$. Then we say that the function f is subordinate to g, if there exist a Schwarz function $w(z)$ which is analytic in $\mathbb{U}$ with $w(0) = 0$, $|w(z)| < 1, (z \in \mathbb{U})$ satisfying $f(z) = g(w(z))$. It is known that, $f(z) \prec g(z)$ and g is univalent $\iff f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$.

By the Koebe one-quarter theorem [2] every function $f \in \mathcal{S}$ has an inverse f^{-1} defined by $f^{-1}(f(z)) = z, (z \in \mathbb{U})$ and $f(f^{-1}(w)) = w, \left(|w| < r_0(f),\ r_0(f) \geq \frac{1}{4}\right)$ where

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - ... \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Denote by Σ the class of bi-univalent functions in $\mathbb{U}$. The familiar Koebe function is not a member of Σ. Examples of bi-univalent functions are $\frac{z}{1-z}, -log(1-z)$, $\frac{1}{2}log\left(\frac{1+z}{1-z}\right)$, ... Lewin [7] inspected the class of bi-univalent function Σ and attained the bound $|a_2| < 1.51$. Brannan and Clunie [1] conjectured that $|a_2| \leq \sqrt{2}$. On the other hand, Netanyahu [8] showed that $max_{f\in\Sigma}|\alpha_2| = \frac{4}{3}$. In 1984, Tan [10] obtained the best known estimate for functions in Σ as $|a_2| < 1.485$. The coefficient estimate problem for $|a_n|$ $(n \in \mathbb{N}, n \geq 3)$ for functions in Σ is still open. Many researchers [5, 11, 12] explored several interesting subclasses of the class Σ and found non-sharp estimates for the first two Taylor-Maclaurin coefficients.

Definition 1.1. (see [3, 4]) *For $n \geq 3$, Horadam polynomial sequence $h_n(r, a, b, p, q)$, or briefly $h_n(r)$, is defined by the recurrence relation*

$$h_n(r) = prh_{n-1}(r) + qh_{n-2}(r) \ \ (r \in \mathbb{R};\ n \in \mathbb{N} = \{1, 2, 3...\}) \tag{1.3}$$

where $h_1(r) = a$ and $h_2(r) = br$, for some real constants a, b, p and q.
The generating function of the Horadam polynomials $h_n(r)$ (see [4]) are given by

$$\Omega(r, z) = \Sigma_{n=1}^{\infty} h_n(r)z^{n-1} = \frac{a + (b - ap)rz}{1 - prz - qz^2}. \tag{1.4}$$

The (p,q)-analogue of Sălăgean differential operator [9] $\mathfrak{T}_{p,q}^k f(z) : \mathcal{A} \to \mathcal{A}$ for $k \in \mathbb{N}$, is formed as follows.

$$\begin{aligned} \mathfrak{T}_{p,q}^0 f(z) &= f(z), \\ \mathfrak{T}_{p,q}^1 f(z) &= z(\mathfrak{T}_{p,q} f(z)), \\ &\ \ . \\ \mathfrak{T}_{p,q}^k f(z) &= z\mathfrak{T}_{p,q}(\mathfrak{T}_{p,q}^{k-1} f(z)). \end{aligned}$$

From the definition of $\mathfrak{T}^k_{p,q}f(z)$ we get

$$\mathfrak{T}^k_{p,q}f(z) = z + \sum_{n=2}^{\infty}[n]^k_{p,q}a_n z^n \ (z \in \mathbb{U}).$$

Definition 1.2. *For $\zeta \geq 1, \ \varrho \geq 0$ and $\delta \geq 0$, a function $f \in \mathcal{A}$ given by (1.1) is said to be in the class $\mathfrak{M}_\zeta(p,q,k,\varrho)$ if the following subordinations are satisfied:*

$$(1-\zeta)\left(\frac{\mathfrak{T}^k_{p,q}f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}^k_{p,q}f(z))'\left(\frac{\mathfrak{T}^k_{p,q}f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}^k_{p,q}f(z))'' \prec \Omega(r,z)+1-a. \tag{1.5}$$

Definition 1.3. *For $\zeta \geq 1, \ \varrho \geq 0$ and $\delta \geq 0$, a function $f \in \Sigma$ given by (1.1) is said to be in the class $\mathfrak{B}_\zeta(p,q,k,\varrho)$ if the following subordinations are satisfied:*

$$(1-\zeta)\left(\frac{\mathfrak{T}^k_{p,q}f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}^k_{p,q}f(z))'\left(\frac{\mathfrak{T}^k_{p,q}f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}^k_{p,q}f(z))'' \prec \Omega(r,z)+1-a \tag{1.6}$$

and

$$(1-\zeta)\left(\frac{\mathfrak{T}^k_{p,q}g(w)}{w}\right)^{\varrho} + \zeta(\mathfrak{T}^k_{p,q}g(w))'\left(\frac{\mathfrak{T}^k_{p,q}g(w)}{w}\right)^{\varrho-1} + \delta w(\mathfrak{T}^k_{p,q}g(w))'' \prec \Omega(r,w)+1-a \tag{1.7}$$

where $g(w) = f^{-1}(w)$ is defined by (1.2).

2. Coefficient bounds for $f \in \mathfrak{M}_\zeta(p,q,k,\varrho)$

Let $\mathcal{B} = \{\omega \in \mathcal{H} : |\omega(z)| \leq 1, z \in \mathbb{U}\}$ and $\mathcal{B}_0$ be the subclass of $\mathcal{B}$ of all ω such that $\omega(0) = 0$. Like it was previously mentioned, the elements of $\mathcal{B}_0$ are known as Schwarz functions.

Lemma 2.1. [6] *If $\omega \in \mathcal{B}_0$ is of the form*

$$\omega(z) = \sum_{n=1}^{\infty}\omega_n z^n, \ \ z \in \mathbb{U}, \tag{2.1}$$

then for $\nu \in \mathbb{C}$,

$$|\omega_2 - \nu\omega_1^2| \leq max\{1, |\nu|\}. \tag{2.2}$$

Theorem 2.1. *Let $f \in \mathfrak{M}_\zeta(p,q,k,\varrho)$. Then $|a_2| \leq \dfrac{|br|}{[2]^k_{p,q}(\varrho+\zeta+2\delta)}$,*

$$|a_3| \leq \frac{|br|}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}} \ max\left\{1, \left|\left(\frac{(\varrho+2\zeta)(\varrho-1)br}{2(\varrho+\zeta+2\delta)^2}\right) - \frac{pbr^2+aq}{br}\right|\right\}$$

$$and\ |a_3 - \varrho a_2^2| \leq \frac{|br|}{(\varrho+2\zeta+6\delta)[3]_{p,q}^k} max\left\{1, \left|\frac{(\varrho+2\zeta)br}{2(\varrho+\zeta+2\delta)^2}\right.\right.$$

$$\left.\left.\left(\frac{2\varrho(\varrho+2\zeta+6\delta)[3]_{p,q}^k}{(\varrho+2\zeta)([2]_{p,q}^k)^2}+\varrho-1\right) - \frac{pbr^2+aq}{br}\right|\right\}.$$

Proof. Let f be in the class $\mathfrak{M}_\zeta(p,q,k,\varrho)$ then from Definition 1.2, for some analytic functions u such that $u(0)=0$ and $|u(z)|<1$, for all $z\in\mathbb{U}$, then we can write

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k f(z))'\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}_{p,q}^k f(z))'' = \Omega(r,u(z))+1-a$$

or equivalently,

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k f(z))'\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}_{p,q}^k f(z))'' = 1+h_1(r)+h_2(r)u(z)+h_3(r)(u(z))^2+...-a \quad (2.3)$$

From the equality (2.3), we have

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k f(z))'\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}_{p,q}^k f(z))'' = 1+h_2(r)u_1(z)+[h_2(r)u_2+h_3(r)u_1^2]z^2+... \quad (2.4)$$

It is well-known that if

$$|\ u(z)\ |=|\ u_1z+u_2z^2+u_3z^3+...\ |<1,\ (z\in\mathbb{U})$$

then

$$|\ u_t\ |\leq 1\ for\ t\in\mathbb{N}. \quad (2.5)$$

Comparing the coefficients of equation (2.4), we get

$$[2]_{p,q}^k(\varrho+\zeta+2\delta)a_2 = h_2(r)u_1 \quad (2.6)$$

$$(\varrho+2\zeta)\left\{\left(\frac{\varrho-1}{2}\right)([2]_{p,q}^k)^2a_2^2+\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^k a_3\right\} = h_2(r)u_2+h_3(r)u_1^2 \quad (2.7)$$

From (2.6) we get,

$$|a_2| \leq \frac{|br|}{[2]_{p,q}^k(\varrho+\zeta+2\delta)}. \quad (2.8)$$

Using (2.8) in (2.7), we get

$$(\varrho+2\zeta)(1+\frac{6\delta}{\varrho+2\zeta})[3]^k_{p,q}a_3 = h_2(r)u_2 + h_3(r)u_1^2 - (\varrho+2\zeta)\left(\frac{\varrho-1}{2}\right)([2]^k_{p,q})^2 a_2^2$$

$$= h_2(r)u_2 - \frac{u_1^2}{2}\left[\left(\frac{h_2(r)(\varrho+2\zeta)(\varrho-1)}{(\varrho+\zeta+2\delta)^2}\right) - 2h_3(r)\right].$$

Thus we have

$$a_3 = \frac{h_2(r)}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}\left\{u_2 - u_1^2\left[\left(\frac{(\varrho+2\zeta)(\varrho-1)h_2(r)}{2(\varrho+\zeta+2\delta)^2}\right) - \frac{h_3(r)}{h_2(r)}\right]\right\}$$

$$= \frac{h_2(r)}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}\{u_2 - \aleph u_1^2\} \tag{2.9}$$

where $\aleph = \left[\left(\frac{(\varrho+2\zeta)(\varrho-1)h_2(r)}{2(\varrho+\zeta+2\delta)^2}\right) - \frac{h_3(r)}{h_2(r)}\right]$. By applying Lemma 2.1, we get

$$|a_3| = \frac{|h_2(r)|}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}|u_2 - \aleph u_1^2|$$

$$\leq \frac{|br|}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}\ max\left\{1, \left|\left(\frac{(\varrho+2\zeta)(\varrho-1)br}{2(\varrho+\zeta+2\delta)^2}\right) - \frac{pbr^2+aq}{br}\right|\right\}.$$

For any $\varrho \in \mathbb{C}$, we get

$$a_3 - \varrho a_2^2 = \frac{h_2(r)}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}\left\{u_2 - u_1^2\left[\left(\frac{(\varrho+2\zeta)(\varrho-1)h_2(r)}{2(\varrho+\zeta+2\delta)^2}\right) - \frac{h_3(r)}{h_2(r)}\right]\right\}$$

$$- \varrho\left(\frac{h_2(r)u_1}{[2]^k_{p,q}(\varrho+\zeta+2\delta)}\right)^2$$

$$= \frac{h_2(r)}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}\{u_2 - \eta u_1^2\}$$

where

$$\eta = \frac{(\varrho+2\zeta)h_2(r)}{2(\varrho+\zeta+2\delta)^2}\left(\frac{2\varrho(\varrho+2\zeta+6\delta)[3]^k_{p,q}}{(\varrho+2\zeta)([2]^k_{p,q})^2} + \varrho - 1\right) - \frac{h_3(r)}{h_2(r)}.$$

By applying Lemma 2.1, we get

$$|a_3 - \varrho a_2^2| = \frac{|h_2(r)|}{(\varrho+2\zeta+6\delta)[3]^k_{p,q}}|u_2 - \eta u_1^2|$$

$$|a_3-\varrho a_2^2| \leq \frac{|br|}{(\varrho+2\zeta+6\delta)[3]_{p,q}^k} max\left\{1, \left|\frac{(\varrho+2\zeta)br}{2(\varrho+\zeta+2\delta)^2}\left(\frac{2\varrho(\varrho+2\zeta+6\delta)[3]_{p,q}^k}{(\varrho+2\zeta)([2]_{p,q}^k)^2}+\varrho-1\right)-\frac{pbr^2+aq}{br}\right|\right\}.$$

Theorem 2.2. *Let f given by (1.1) be in the class $\mathfrak{B}_\zeta(p,q,k,\varrho)$. Then*
$$| a_2 |\leq \frac{| br | \sqrt{2 | br |}}{\sqrt{|\Theta(\varrho,\zeta,p,q,k)|}} \text{ and } |a_3| \leq \frac{b^2r^2}{([2]_{p,q}^k)^2(\varrho+\zeta+2\delta)^2}+\frac{|br|}{(\varrho+2\zeta)(1+\frac{6\delta}{2\zeta+1})[3]_{p,q}^k}$$
where

$$\Theta(\varrho,\zeta,p,q,k)=\{(\varrho+2\zeta)[(\varrho-1)([2]_{p,q}^k)^2+2(1+\frac{6\delta}{\varrho+2\zeta})[3]_{p,q}^k]b - 2([2]_{p,q}^k(\varrho+\zeta+2\delta))^2p\}br^2-2[[2]_{p,q}^k(\varrho+\zeta+2\delta)^2]aq. \tag{2.10}$$

Proof. Let f is in the class $\mathfrak{B}_\zeta(p,q,k,\varrho)$ then from Definition 1.3, for some analytic functions u and v such that $u(0)=v(0)=0$ and $|u(z)|<1$, $|v(w)|<1$ for all $z,w\in\mathbb{U}$, we can write

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k f(z))'\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}_{p,q}^k f(z))''=\Omega(r,u(z))+1-a$$

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k g(w)}{w}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k g(w))'\left(\frac{\mathfrak{T}_{p,q}^k g(w)}{w}\right)^{\varrho-1} + \delta w(\mathfrak{T}_{p,q}^k g(w))''=\Omega(r,v(w))+1-a$$

or equivalently,

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k f(z))'\left(\frac{\mathfrak{T}_{p,q}^k f(z)}{z}\right)^{\varrho-1} + \delta z(\mathfrak{T}_{p,q}^k f(z))''= 1+h_2(r)u_1(z)+[h_2(r)u_2+h_3(r)u_1^2]z^2+... \tag{2.11}$$

$$(1-\zeta)\left(\frac{\mathfrak{T}_{p,q}^k g(w)}{w}\right)^{\varrho} + \zeta(\mathfrak{T}_{p,q}^k g(w))'\left(\frac{\mathfrak{T}_{p,q}^k g(w)}{w}\right)^{\varrho-1} + \delta w(\mathfrak{T}_{p,q}^k g(w))''= 1+h_2(r)v_1(w)+[h_2(r)v_2+h_3(r)v_1^2]w^2+... \tag{2.12}$$

It is well-known that if $| u(z) |=| u_1 z + u_2 z^2 + u_3 z^3 + ... |< 1$, $(z \in \mathbb{U})$ and $| v(w) |=| v_1 w + v_2 w^2 + v_3 w^3 + ... |< 1$, $(w \in \mathbb{U})$ then

$$| u_t |\leq 1 \ and \ \ | v_t |\leq 1 \ for \ t \in \mathbb{N}. \tag{2.13}$$

Comparing the coefficients of equation (2.11) and (2.12), we obtain

$$[2]_{p,q}^k(\varrho+\zeta+2\delta)a_2 = h_2(r)u_1 \tag{2.14}$$

$$(\varrho+2\zeta)\left\{\left(\frac{\varrho-1}{2}\right)([2]_{p,q}^k)^2 a_2^2 + \left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^k a_3\right\} = h_2(r)u_2+h_3(r)u_1^2 \tag{2.15}$$

$$-[2]_{p,q}^k(\varrho+\zeta+2\delta)a_2 = h_2(r)v_1 \tag{2.16}$$

$$(\varrho+2\zeta)\left\{\left(\frac{\varrho-1}{2}\right)([2]_{p,q}^k)^2 a_2^2 + \left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^k(2a_2^2-a_3)\right\} = h_2(r)v_2 + h_3(r)v_1^2 \tag{2.17}$$

From (2.14) and (2.16) we obtain,

$$u_1 = -v_1 \tag{2.18}$$

$$2\{[2]_{p,q}^k(\varrho+\zeta+2\delta)\}^2 a_2^2 = h_2^2(r)(u_1^2+v_1^2) \tag{2.19}$$

Adding (2.15) and (2.17) we obtain,

$$2(\varrho+2\zeta)\left\{\frac{\varrho-1}{2}([2]_{p,q}^k)^2 + \left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^k\right\}a_2^2 = h_2(r)(u_2+v_2)+h_3(r)(u_1^2+v_1^2) \tag{2.20}$$

Substituting the value of $(u_1^2+v_1^2)$ from (2.19) in the right hand side of (2.20) we obtain,

$$a_2^2 = \frac{h_2^3(r)(u_2+v_2)}{(\varrho+2\zeta)\left\{(\varrho-1)([2]_{p,q}^k)^2 + 2\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^k\right\}h_2^2(r) - 2h_3(r)([2]_{p,q}^k(\varrho+\zeta+2\delta))^2} \tag{2.21}$$

Using (1.3), (2.10), (2.13) and (2.21), we obtain

$$| a_2 |\leq \frac{| br | \sqrt{2 \mid br \mid}}{\sqrt{|\Theta(\varrho,\zeta,p,q,k)|}}.$$

Subtracting (2.17) from (2.15) we obtain,

$$2(\varrho+2\zeta)\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^{k}(a_3-a_2^2)=h_2(r)(u_2-v_2). \tag{2.22}$$

In view of (2.19) and (2.21), Equation (2.22) becomes

$$a_3=\frac{h_2^2(r)(u_1^2+v_1^2)}{2([2]_{p,q}^{k}(\varrho+\zeta+2\delta))^2}+\frac{h_2(r)(u_2-v_2)}{2(\varrho+2\zeta)\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^{k}}$$

By applying (1.3), we get,

$$|a_3|\leq\frac{b^2r^2}{([2]_{p,q}^{k})^2(\varrho+\zeta+2\delta)^2}+\frac{|br|}{(\varrho+2\zeta)(1+\frac{6\delta}{\varrho+2\zeta})[3]_{p,q}^{k}}.$$

By setting $\varrho=\delta=0$ and $\zeta=1$ in Theorem 2.2, we obtain the following Corollary.

Corollary 2.1. *If f of the form (1.1) is in the class $\mathfrak{B}_1(p,q,k)$ then*
$$\mid a_2\mid\leq\frac{\mid br\mid\sqrt{\mid br\mid}}{\sqrt{|\{(2[3]_{p,q}^{k}-([2]_{p,q}^{k})^2)b-[2]_{p,q}^{k}p\}br^2-[2]_{p,q}^{k}aq|}}\ \text{and}\mid a_3\mid\leq\frac{b^2r^2}{([2]_{p,q}^{k})^2}+\frac{\mid br\mid}{2[3]_{p,q}^{k}}.$$
Setting $\varrho=\delta=0$, $\zeta=1$ and $k=0$ in Theorem 2.2, we obtain

Corollary 2.2. *If f of the form (1.1) is in the class $\mathfrak{B}_1(r)$ then*
$$\mid a_2\mid\leq\frac{\mid br\mid\sqrt{\mid br\mid}}{\sqrt{|\{b-p\}br^2-aq|}}\ \text{and}\mid a_3\mid\leq b^2r^2+\frac{\mid br\mid}{2}.$$

3. Fekete-Szegö inequality for the class $\mathfrak{B}_\zeta(p,q,k,\varrho)$

Theorem 3.1. *Let f given by (1.1) be in the class $\mathfrak{B}_\zeta(p,q,k,\varrho)$ and $\mu\in\mathcal{R}$. Then*

$$\mid a_3-\varrho a_2^2\mid\leq\begin{cases}\frac{2|br|}{2(\varrho+2\zeta)\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^{k}}, & 0\leq\mid\phi(\varrho,r)\mid\leq\frac{1}{2(\varrho+2\zeta)\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^{k}}\\ 2|br||\phi(\varrho,r)|, & |\phi(\varrho,r)|\geq\frac{1}{2(\varrho+2\zeta)\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^{k}}\end{cases}$$

where $\phi(\varrho,r)=\dfrac{h_2^2(r)(1-\varrho)}{\Upsilon(p,q,k,\varrho)}$ *and*
$$\Upsilon(p,q,k,\varrho)=(\varrho+2\zeta)\left\{(\varrho-1)([2]_{p,q}^{k})^2+2\left(1+\frac{6\delta}{\varrho+2\zeta}\right)[3]_{p,q}^{k}\right\}$$
$$h_2^2(r)-2h_3(r)([2]_{p,q}^{k}(\varrho+\zeta+2\delta))^2.$$

4. Conclusion

In the present work, by making use of Sălăgean differential operator, we define a new subclass of analytic and bi-univalent functions using the Horadam polynomial. Coefficient estimate $|a_2|$, $|a_3|$ and Fekete-Szegö inequality of the functions has been studied.

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